Physically-based model for gully simulation: application to the Brazilian Semiarid Region - Supplement

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1 Rain data

Daily rainfall data from the Foundation of Meteorology and Water Resources of Ceará (FUNCEME) were used in the study. The interval of the study was nearly sixty years and to cover the whole period, data from three stations were used:

1. From 1958 to 1973: Station Coroatá (5.03º S; 39.33º W)
2. From 1974 to 1987: Station Boa Viagem (5.12º S; 39.73º W)
3. From 1988 onward: Station Madalena (4.85º S; 39.57º W)
4. Validation: Station Uruquê (5.14º S; 39.18º W)
5. Validation: Station Paus Ferro (4.99º S; 39.48º W)

All stations were equipped with Pluviometer Ville de Paris with a standard opening of 400 cm^2. The final rain series is presented in Figure S1. To assess the quality of the data series, double mass analysis with other stations in the vicinity was performed, as ilustrated in Figure S2, from which we can see that there is no relevant bias in the data.

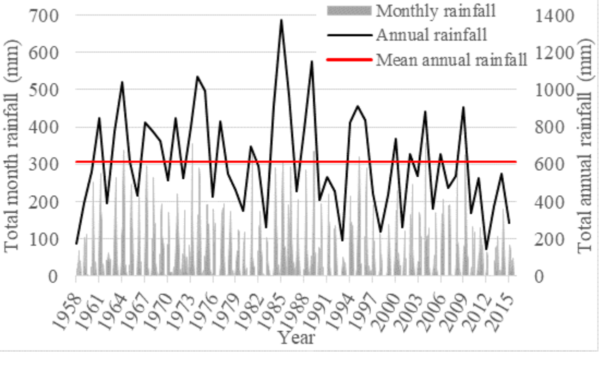

Figure S1. Rain series with monthly (left vertical axes) and yearly (right vertical axes) rainfall. The red line indicates the average anual rainfall (over 600 mm yr^{-1})

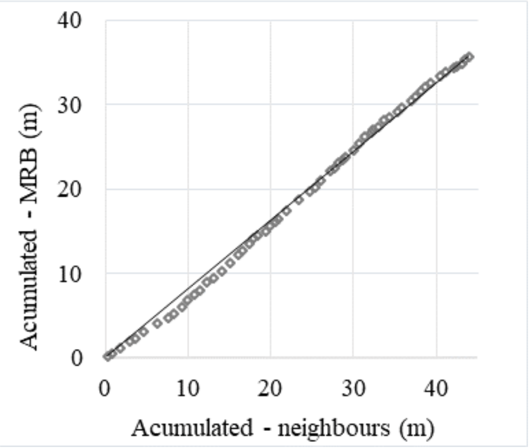

Figure S2. Double mass diagram. Neighbour stations in a radius of 50 km.

2 Gully models from literature

2.1 Foster and Lane Model (FLM)

The Foster and Lane (1983) ephemeral-gully model aims at explaining "erosion by concentrated flow in farm fields" for single runoff intensive events. The gullies are considered ephemeral as productive farmlands usually provide periodic tillage to diminish or remove the gullies generated by previous events. The model is physically based and uses the Manning equation, mass balance, and shear stress mobilisation; it assumes an equilibrium channel width and the gully evolution in two steps. The first step is the vertical incision when the concentrated overflow starts digging the channel with a constant width. The second step starts after the bottom of the channel reaches a non-erodible layer. Then, the section starts a sideward erosive process, widening until the end of the effective runoff, i.e., with a shear stress below the critical stress. Detachment ratio (D_r) and shear stress (τ) are given by the Equations (1) and (2).

$$D_r = K_r\left(\tau - \tau_c\right) \tag{1}$$

$$\tau(X) = 1.35\gamma R_h S\left[1 - \left(1 - 2\frac{X}{WP}\right)^{2.9}\right] \tag{2}$$

In Equation 2, X is the position of a point on the channel bed, varying from zero to WP (wet perimeter). S is the longitudinal slope of the channel; r_h the hydraulic radius; and γ is the specific gravity of water (assumed 9.81 kN m^{-3}).

In order to model long-term gullies using Foster and Lane (1983) equations, the following assumptions were made: First, all mobilised sediment is carried away by the discharges, i.e., there is no sediment deposition on the channel bed. This assumption was confirmed by field surveys in many sections where very little loose sediment was identified. Secondly, in the long run, the effect of each intense runoff can be piled in a cumulative model of widths/depths layers. This implies that each erosive event does not suffer significant influence of the previous, and the total eroded soil is related only with the energy of each event. The piling process considered all events with runoff. The following figures display the flow charts for the original Foster and Lane Model and how it allows to model multiple events by piling area available in the supplementary material (Fig. S3 and S4).

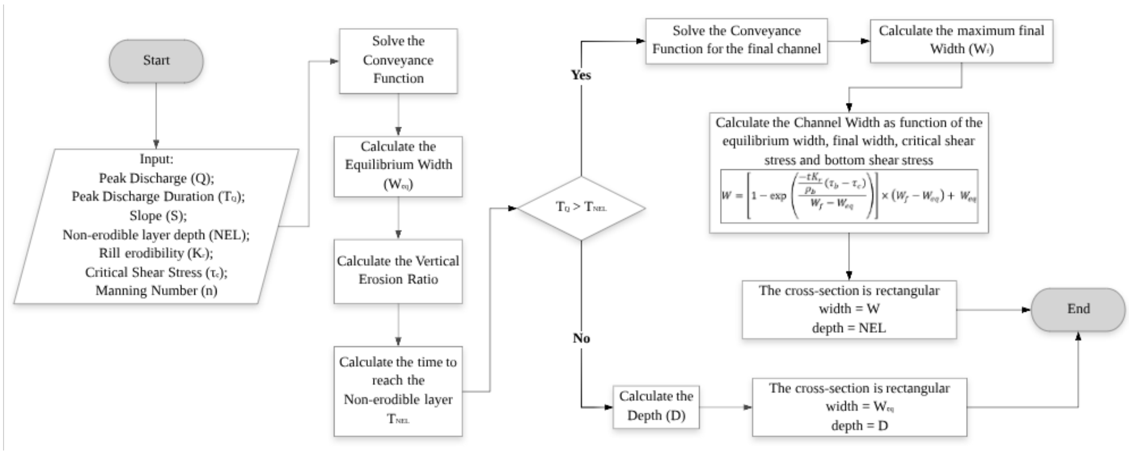

Figure S3. Flowchart of the original Foster and Lane Model (1983)

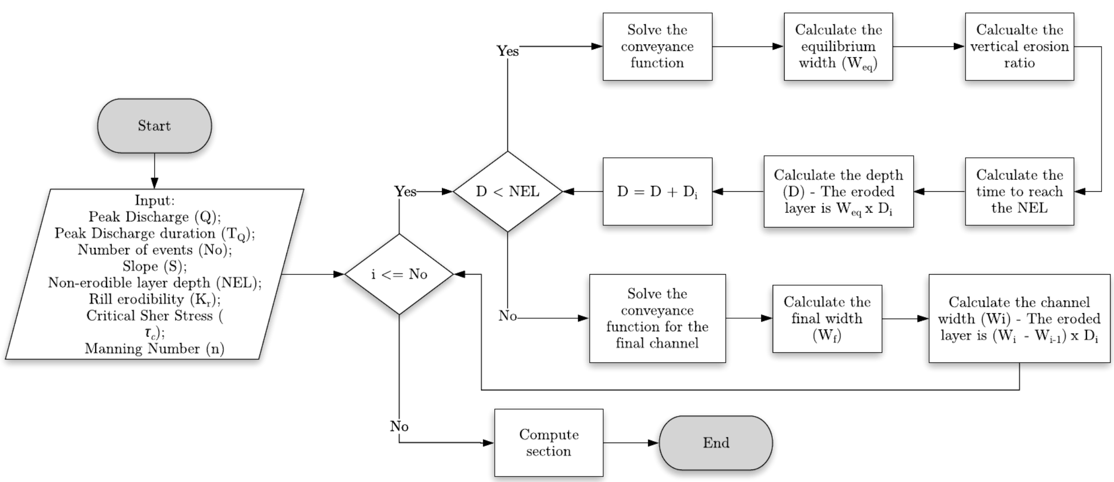

Figure S4. Flowchart of the modified Foster and Lane Model, in order to allow multiple successive events.

2.2 Sidorchuk Model (SM)

The Sidorchuk model (Sidorchuk, 1999) is both physically and empirically based. It considers mass balance of sediment, shear stress (in terms of critical velocity), soil cohesion and the Manning equation to estimate the cross-section geometry and channel slope. It also uses empirical equations based on field measurement to estimate the flow depth and width. The model gives special attention to the processes involving gully wall transformation, as shown in Equations (3) and (4).

$$D_{vcr} = \frac{2C_h}{g\ \rho_s} \cos(\varphi) \sin^{-2}\left[\frac{1}{2}\left(\varphi + \frac{\pi}{2}\right)\right] \tag{3}$$

$$\frac{C_h}{g\ \rho_s\ D_v} = \frac{\rho_s - w\rho}{\rho} \tan\varphi \cos^2\phi - \frac{\sin 2\phi}{2} \tag{4}$$

In Equations (3) and (4), C_h is soil cohesion; D_v the depth incision; D_{vcr} the critical value of depth for wall failure; w the volumetric soil water content; ρ_s the bulk density of the soil; ρ the density of water; g gravity's acceleration; φ the soil internal friction angle; and ϕ the wall slope, in degrees. A flow chart of the model is available in the supplementary material (Fig. S5).

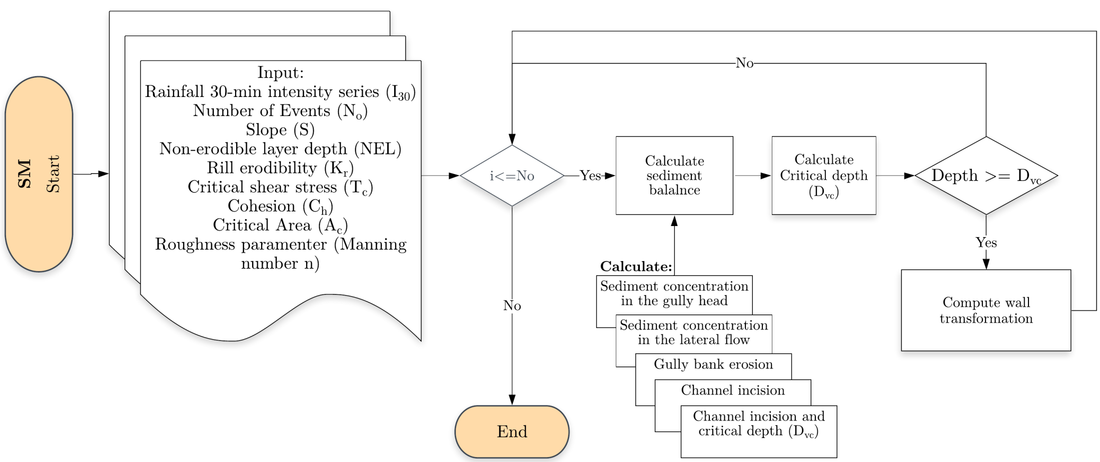

Figure S5. Flowchart of the Sidorchuk Model (1999).

3 Model evaluators

To allow us to assess a more realistic, concerning uncertainty, evaluation of the model's quality, we implemented the routine proposed by Ritter and Muñoz-Carpena (2013). The routine, called FITEVAL, allows the modeller to obtain a confidence interval for the Nash-Sutcliffe Efficiency (NSE). The common values of NSE is, therefore, interpreted as the centre of this interval and of the probabilistic distribution of its values depending on how the data are resampled. The method classifies the model as Acceptable to Very good – NSE $\in$ [0.66, 0.95]; (p-value = 0.05). A conservative interpretation of these results is to understand the lowest values as the minimum state of information, i.e., the one that contains (almost) no unproved hypothesis and does not rely on luck. Therefore, the model can be classified, in the worst case scenario, as Acceptable. The output of the

FITEVAL is presented in (Figure S6). On the left we see a scatter of points Computed (modelled) and Observed (measured) points, corresponding to the cross-section areas. Below, we have a list of the percentiles of each category. On the right we have the cumulative probabilistic distribution of values of efficiency and below that, a plot showing the observed (diamonds) and computed (continuous line) in decreasing order.

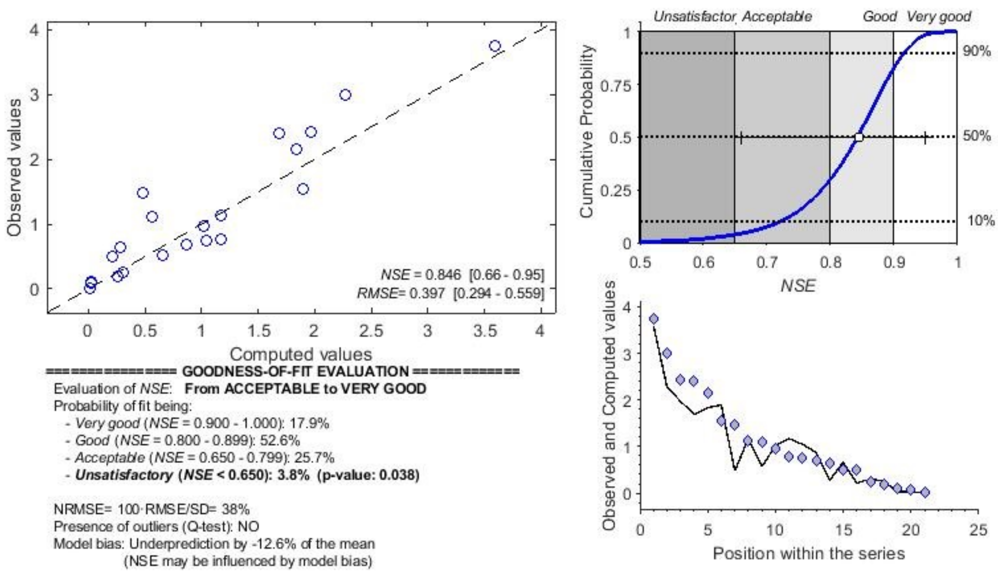

Figure S6. Output of FITEVAL (Ritter and Muñoz-Carpena, 2013), indicating the model proposed as, in the worst case scenario, acceptable. On the left we have a scatter plot of modelled and measured data, on the right the cumulative probability distribution of NSE (top) and the values of measured (black line) and modelled (diamond) in decreasing order (bottom).

4 General data and code

Following we present basic data and results used in the model and the code used. For more, please access the following link for the permanent repository:

github.com/PedroAlencarTUB/GullyModel-FLSM

Gully	Section	Basin Area (m^2)	Perimeter (m)	Length (m)	Kc	Kf	Conc. Time (s)	Measured			
								Area (m^2)	Perimeter (m)	Width (m)	Prof
Gully 1	S1	3200	432.5	195	2.141	0.084	25	0.248	2.322	2.264	0.215
	S2	3080	415.5	187.8	2.096	0.087	23	0.742	3.451	3.188	0.582
	S3	1183	337.4	179.3	2.747	0.037	22	0.684	3.734	3.558	0.498
	S4	491	107.7	35.9	1.361	0.381	6	0.505	3.826	3.763	0.288
	S5	78	49.8	22.9	1.579	0.149	5	0.084	1.520	1.498	0.103
	S6	3060	408	184.5	2.065	0.090	23	0.770	2.707	2.180	0.709
Gully 2	S1	2000	383	123	2.398	0.132	31	0.721	2.235	1.853	0.479
	S2	1870	379	114.3	2.454	0.143	29	2.428	7.559	7.322	0.694
	S3	1470	336	99.7	2.454	0.148	28	1.105	6.950	6.891	0.307
	S4	252	79.9	23.5	1.409	0.456	7	0.644	3.085	2.951	0.356
	S5	40	43.7	12.4	1.935	0.260	4	0.008	0.488	0.479	0.033
	S6	1930	381	119	2.428	0.136	30	2.403	7.633	7.438	0.653
	S7	1425	320	92.2	2.374	0.168	25	1.470	6.263	6.163	0.451
Gully 3	S1	3500	554	204	2.622	0.084	40	1.126	6.459	6.321	0.521
	S2	3400	540	193	2.593	0.091	39	0.962	5.596	5.543	0.306
	S3	1000	248	105	2.196	0.091	25	0.200	2.239	2.181	0.198
	S4	56	49.1	13.9	1.837	0.290	4	0.103	1.351	1.303	0.117
	S5	2230	473	185.7	2.805	0.065	38	0.511	3.098	3.016	0.306
	S6	4800	620	213	2.506	0.106	43	3.751	6.485	6.057	0.897
	S7	4964	639	218	2.539	0.104	45	2.151	5.744	4.679	0.785
	S8	4712	609	205	2.484	0.112	41	1.543	5.935	5.327	0.844

Table S1. Paper data compilation part 1 - Topography results

Gully	FLM-Iav		FLM-I60		FLM-I30		FLM-I15		FLM-	SM	FL-SM
	Area (m²)	Width (m)	Area (m²)	Width (m)	Area (m²)	Width (m)	Area (m²)	Width (m)	Area (m²)	Area (m²)	Area (m²)
Gully 1	0.071	0.299	0.208	0.970	0.301	1.193	0.288	1.309	0.306	0.380	0.301
	0.283	0.476	0.862	1.423	1.047	1.720	1.106	1.877	1.094	1.685	1.047
	0.165	0.331	0.500	0.992	0.871	1.215	0.866	1.325	0.494	1.118	0.871
	0.050	0.173	0.172	0.542	0.216	0.663	0.210	0.726	0.641	0.388	0.216
	0.006	0.061	0.023	0.222	0.028	0.274	0.039	0.298	0.043	0.082	0.028
	-	-	-	-	1.170	1.669	-	-	1.248	2.173	1.170
Gully 2	0.207	0.431	0.624	1.303	1.359	1.592	1.142	1.738	2.324	2.276	2.276
	0.278	0.401	0.855	1.199	1.369	1.467	1.474	1.556	2.029	1.968	1.968
	0.100	0.326	0.315	0.984	0.567	1.206	0.505	1.318	0.860	0.590	0.567
	0.049	0.137	0.153	0.431	0.279	0.532	0.211	0.581	0.940	0.473	0.279
	0.002	0.060	0.005	0.154	0.006	0.187	0.007	0.203	0.014	0.006	0.006
	-	-	-	-	1.202	1.287	-	-	1.721	1.690	1.690
	-	-	-	-	0.477	1.081	-	-	0.791	0.917	0.477
Gully 3	0.328	0.641	0.961	1.877	1.171	2.288	2.003	2.493	1.192	1.752	1.171
	0.160	0.522	0.478	1.562	1.021	1.912	0.914	2.086	1.101	0.803	1.021
	0.048	0.242	0.152	0.740	0.254	0.909	0.234	0.991	0.273	0.289	0.254
	0.008	0.067	0.022	0.189	0.028	0.239	0.031	0.268	0.068	0.086	0.028
	0.120	0.392	0.379	1.183	0.660	1.450	0.582	1.584	0.558	0.661	0.660
	-	-	-	-	2.193	2.275	-	-	2.626	3.596	3.596
	-	-	-	-	1.832	2.318	-	-	2.173	3.025	1.832
	-	-	-	-	1.891	2.250	-	-	2.359	3.279	1.891

Table S2. Paper data compilation part 2 - Model results

Gully	Volume (Total Station)	Volume (UAV)	Error volume	Length	Max. Width	Max. Depth	Channel area	Coordenates UTM (zone: 23S)	
	(m³)	(m³)	(%)	(m)	(m)	(m)	(m²)	X (m)	Y (m)
1	20	22.5	11%	45.5	3.7	0.709	150	445297	9449332
2	38	33.5	13%	33.1	5.8	0.694	320	444614	9447240
3	69	-	-	37.4	8.4	0.844	480	444883	9447479

Table S3. Comparison of gullies dimensions.

Land-use description	Condition	Soil Class			
		A	B	C	D
Cultuivated land	with conservation treatment	62	71	78	81
	without conservation treatment	72	81	88	91
Pasture	good	39	61	74	80
	bad	68	79	(86)	89

Table S4. CN values by land-use (Chow et al., 1988). In parentesis the CN adopted in this study.

4.1 Fortran Code

```
program FLSM

IMPLICIT NONE
CHARACTER arquivo*30,arquivo1*30,arquivo2*30
integer No,i
REAL Q(1500),n,S,Kr,Ds,Pa,Tc,Xnc,Gc,Rn,WP,Rh,Ta,Wn,Weq,Le,Delta,Param,Tmax,tm(1500),t(1500),tn
REAL NEL,Tb,Gcf,Xncf,Xnc1,Xnc2,Gc1,Gc2,Rn1,Rn2,WP1,Lim,Ch,Phi,Ang,Dvcr,g,pi,es,Ws
REAL WP2,Rh1,Rh2,Ta1,Ta2,Dg1,Dg2,Tnc1,Tnc2,Xncf1,Xncf2,gcf1,gcf2,tnf1,tnf2,dgf1,dgf2
REAL wi(1500),wf,dp(1500),depth,DeltaW,PerimF,AreaI,AreaF,Rh_S,Max_wi,Max_W,dW
COMMON /grand1/Gcf,Xncf1,Xncf2
COMMON /grand2/Ch,g,Ds,es,Pa,Phi,depth,Ang,pi

!Phi = internal friction angle of the soil (degrees) > the data can be obtained in the NAVFAC 7.02,
!based in texture; in Ortiz et al 1989 based in texture, Plasticity intex and porosity or
!via laboratory experiments
!Ch = Cohesion (Pa) > the data can be obtained in the NAVFAC 7.02, based in texture;
!in Ortiz et al 1989 based in texture, Plasticity intex and porosity or via laboratory experiments
!Lim = a threshold for aplication of sidorchuk routine
!Q = peak discharge (m3/s)
!tm  = discharge's duration (min)
!n = Manning's number
!S = Slope (abs) - (not in degrees or percentage)
```

```
!Kr = rill erodibility factor (s/m)
!Tc = Critical shear stress (Pa)
!Xnc = Normalized distance in the WP where T=Tc (abs)
!Gc = Conveyance function at Xnc (abs)
!Rn = Normalized Hydraulic Radius (abs)
!WP = Wet Perimeter (m)
!Rh = Hydraulic Radius (m)
!Ta = Average shear stress (Pa)
!Wn = Normalized width (abs)
!Weq = equilibrium width (m)
!Er = Erosion rate (kg/m.s)
!Ve = Velocity of movement down (m/s)
!Pse = Weigth of soil eroded in the event (kg)
!Le = Sheet thickness of eroded soil(m)
!Param = A parameter that is repeated throughout the calculations
!t = time (s)
!NEL = Depth of nonerodible layer(m)
!Tb = Tension when the erosion reach the NEL (Pa)
!Gcf = conveyance funcion final (abs)
!Wf = final width of the channel (m)
!tne = time to reach  the NEL (s)

        WRITE(*,*)'*****************************************'
        WRITE(*,*)'           Foster_Lane program           '
        WRITE(*,*)'                                         '
        WRITE(*,*)'    Calculates the final total of soil   '
        WRITE(*,*)'       eroded in an ephemeral gully      '
        WRITE(*,*)'        for a time-serie rainfall        '
        WRITE(*,*)'                                         '
        WRITE(*,*)'      Universidade Federal do Ceara      '
        WRITE(*,*)'    Departamento de Engenharia Agricola  '
        WRITE(*,*)'      Doutorado em Engenharia Agricola    '
        WRITE(*,*)'                                         '
        WRITE(*,*)'            Pedro Alencar, 2019          '
        WRITE(*,*)'                                         '
        WRITE(*,*)'*****************************************'

No = 947 !Number of events

!1. READS INPUT DATA
!write(*,*)'Insert the number of observed events'
!Read(*,*)No
write(*,'(a)')'Insert the name of the file containing the runoff data (Discharge e Duration):
' !in the absense of measured data we sugest use the 30-min intensity
```

```
Read(*,'(a30)')arquivo1
write(*,'(a)')'Insert the name of the file containing the hillslope and soil data: '
Read(*,'(a30)')arquivo2
WRITE(*,'(a)',advance='no') 'Insert the name of the output file: '
READ(*,'(a30)')arquivo
open(50,file=arquivo1)
    do i=1, No
        read(50,*) Q(i),tm(i)
        write(*,*) Q(i),tm(i)
        t(i)=tm(i)*60.
    end do
close(50)
open(60,file=arquivo2)
    read(60,*)n,S,Tc,kr,Ds,es,NEL,Ch,Phi,Lim
    write(*,*)'The number of Manning of the channel is....',n
    write(*,*)'The declivity of the hillslope is.........',S
    write(*,*)'The critical shear stress is..............',Tc
    write(*,*)'The rill erodibility coefficient is........',Kr
    write(*,*)'The soil Bulk density is..................',Ds
    write(*,*)'The porosity of the soil is...............',es
    write(*,*)'The depth of the nonerodible layer is......',NEL
    write(*,*)'The soil cohesion is......................',Ch
    write(*,*)'The internal friction angle is............',Phi
    write(*,*)'The threshold for wall erosion is.........',Lim
close(60)
Pa = 9803. !N/m³
g = 9.803 !gravity
i = 1
depth = 0
AreaI = 0
Max_wi = 0
Max_W  = 0
pi = 3.1415926536
Phi = Phi*pi/180.
Ws = 0.
i=1

!2. FOSTER AND LANE EQUATIONS FOR INCISION BEFORE REACHING THE NEL
open(40,file=arquivo)
do while(i.le.No)
    Param = (n*Q(i)/(S**(0.5)))**0.375

    if (depth.lt.NEL) then
        Gc = Pa*S*Param/Tc
```

```
      if (Gc.lt.1.79) then
          i=i+1
          else

              Delta = sqrt(3.9429**2 - 4*6.9594*(1/Gc))
              Xnc1 = (3.9429-Delta)/(2*6.9594)
              Xnc2 = (3.9429+Delta)/(2*6.9594)
              Rn1 = -0.8834*Xnc1+0.1395*Xnc1+0.151
              Rn2 = -0.8834*Xnc2+0.1395*Xnc2+0.151
      end if
      if (Rn1.gt.0) then
          WP1 = Param/Rn1**(0.625)
          Rh1 = Rn1*WP1
          Ta1 = Pa*Rh1*S
          Tnc1 = Tc/Ta1
          Gc1 = 1./(Tnc1*(Rn1**0.375))
          Dg1 = abs(Gc-Gc1)
          else
              Gc1 = 0
              Dg1 = abs(Gc-Gc1)
      end if
      if (Rn2.gt.0) then
          WP2 = Param/Rn2**(0.625)
          Rh2 = Rn2*WP2
          Ta2 = Pa*Rh2*S
          Tnc2 = Tc/Ta2
          Gc2 = 1/(Tnc2*(Rn2**(0.375)))
          Dg2 = abs(Gc-Gc2)
          else
              Gc2 = 0
              Dg2 = abs(Gc-Gc2)
      end if
      if (Dg1.gt.Dg2) then
          Xnc = Xnc2
          else
              Xnc = Xnc1
      end if
      Write(*,*)'Xnc ',Xnc
      Wn = -1.4873*Xnc+0.7436
      Rn = -0.8834*Xnc**2+0.1395*Xnc+0.151
      WP = Param/Rn**(0.625)
      Weq = WP*Wn
      Rh = Rn*WP
      Ta = Pa*Rh*S
```

```
        Tmax = 1.35*Ta
        if (Tmax.lt.Tc) then
            Write(*,*)'The event didn´t cause erosion.'
            wi(i) = 0.
            dp(i) = 0.
            else
                wi(i) = Weq
                Le = Kr*(Tmax-Tc)*t(i)/Ds
                if (Le.gt.NEL) then
                    dp(i) = NEL
                    else
                        dp(i) = Le
                end if
                depth = depth + dp(i)
                if (Max_wi.lt.Weq) then
                    Max_wi = wi(i)
                    Max_W  = wi(i)
                end if
                AreaI = AreaI+wi(i)*dp(i)
            Write(40,*)'Fase1',i,wi(i),dp(i),Max_wi,AreaI
        end if
    end if

!3. FOSTER AND LANE EQUATIONS FOR INCISION AFTER REACHING THE NEL
    if (depth.ge.NEL) then
        Gcf = Pa*S*Param/Tc
        if (Gcf.lt.1.78) then
            i=i+1
            else
                CALL Newton_NEL
        end if
        tnf1 = 1.35*(1-(1-2.*xncf1)**2.9)
        gcf1 = 1/(tnf1*(xncf1*(1-2*xncf1))**(0.375))
        dgf1 = abs(gcf-gcf1)
        tnf2 = 1.35*(1-(1-2.*xncf2)**2.9)
        gcf2 = 1/(tnf2*(xncf2*(1-2.*xncf2))**(0.375))
        dgf2 = abs(gcf-gcf2)
            if (dgf1.gt.dgf2) then
                xncf = xncf2
                else
                    xncf = xncf1
            end if
        Tb = 1.35*Ta*(1-(1-2*xncf)**2.9)*Pa*S*Param*(xncf*(1-2*xncf))**0.375
        if (Tb.le.Tc)then
```

```
            dW = 0
            else
            dW = Kr*(Tb-Tc)/Ds
        end if

        wf = Param*((1-2.*xncf)/(xncf**1.667))**0.375
        if (Max_W.ge.Wf) then
            tn = 0.
            else
                tn = t(i)*dW/(Wf-Max_W)
        end if
        Max_W = (1-exp(-tn))*(Wf-Max_W)+Max_W
        write(40,*)'Fase2',i, Max_W, NEL, dw
    end if
    i = i+1
end do
DeltaW = Max_W-Max_Wi
AreaF = AreaI + DeltaW*depth
PerimF = Max_W + 2.*depth
Rh_S = AreaF/PerimF

!4. WALL TRANSFORMATION
if (AreaF.gt.Lim) then
    Dvcr = (2.*Ch*cos(Phi)/(g*Ds))/(sin(0.5*(Phi+pi/2.)))**2.
    if  (Dvcr.le.NEL) then
        CALL Newton_PHI
        Ws = Max_W +2.*NEL/tan(Ang)
        AreaF = NEL*(Max_W+Ws)/2.
    end if
end if
write(40,*)'The cross section area is: ',AreaF,' m2'
if (depth.lt.NEL) then
    write(40,*)'The depth is: ',depth,'m'
    write(40,*)' '
    write(40,*)'The erosion didn´t reach the non erodible layer.'
    else
        write(40,*)'The depth is: ',NEL,'m'
        write(40,*)' '
        write(40,*)'The erosion reached the non erodible layer.'
    end if
if (Ws.gt.0.) then
    write(40,*)'There is erosion of the walls.'
    write(40,*)'The top width is',Ws,'m and the wall slope is',100.*tan(Ang),'%'
    write(40,*)Ang*180/pi
```

```
end if
write(*,*)' '
write(*,*)'Please, see the output file'

end program

Subroutine Newton_NEL
!Subrotina para calcular as raizes da equação de Xncf
implicit none
integer i,j
real x,xa,xb,f,df,erro,gcf,xncf1,xncf2
COMMON /grand1/Gcf,Xncf1,Xncf2
Xa = 0.1
Xb = 0.4
i=0
j=0
erro=1000000.
X=Xa
do while (erro.gt.0.000001)
    f = -321.29*x**6+249.02*x**5+9.059*x**4-58.033*x**3+13.012*x**2+1.7199*x-0.004883*COS(x) -1/Gcf
    df = -321.29*6*x**5+249.02*5*x**4+9.059*4*x**3-58.033*3*x**2+13.012*2*x+1.7199+0.004883*sin(x)

    x = xa - f/df
    erro = abs(x-xa)/xa
    xa=x
    i=i+1
end do
erro=1000000.
x=Xb
do while (erro.gt.0.000001)
    f = -321.29*x**6+249.02*x**5+9.059*x**4-58.033*x**3+13.012*x**2+1.7199*x-0.004883*COS(x) -1/Gcf
    df = -321.29*6*x**5+249.02*5*x**4+9.059*4*x**3-58.033*3*x**2+13.012*2*x+1.7199+0.004883*sin(x)

    x = xb - f/df
    erro = abs(x-xb)/xb
    xb=x
    j=j+1
end do
Xncf1 = xa
Xncf2 = xb
end subroutine

Subroutine Newton_PHI
!Subroutine to calculate wall stable angle
```

```
    implicit none

    integer i
    real k1,k2,f1,df1,x,xa,erro,Ch,g,Ds,es,Pa,Phi,depth,Ang,pi
    COMMON /grand2/Ch,g,Ds,es,Pa,Phi,depth,Ang,pi

    i=0
    k1 = Ch/(g*Ds*depth)
    k2 = tan(Phi)*(Ds - es*1000.)/1000.

    erro=1000000.
    x = pi/4.
        do while (erro.gt.0.00000001)

        f1 = k2*(cos(x))**2 - sin(2*x)/2. - k1
        df1 = -2*k2*cos(x) - cos(2*x)

        xa = x - f1/df1
        erro = abs(x-xa)
        x=xa
        i=i+1
    end do
Ang = x
end subroutine
```

Code and data availability. Code and data available in the link: https://github.com/PedroAlencarTUB/GullyModel-FLSM

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